Robust Synchronization of a Class of Robot Manipulators

Manfred Giljum and Gualberto Solís-Perales *

Faculty of Mechanical Engineering, University of Stuttgart,
70550 Stuttgart, Germany
Departamento de Electrónica, CUCEI, U. de G.,
Av. Revolución 1500, Guadalajara Jalisco, México

(Paper received on February 29, 2008, accepted on April 15, 2008)

Abstract. In this work, an adaptive control strategy for the synchronization of robotic manipulators is presented and verified with numerical results. The idea of synchronization is that various systems, that may have completely different dynamics, behave in a way that there exist no residual difference of their outputs. Here we present an approach for the synchronization of robot manipulators in spite of unmodeled dynamics and parametric uncertainties, external disturbances as well as parametric and structural differences of the robots. It is achieved with the help of a nonlinear controller with robust characteristics that only requires the measurement of the angular positions. The uncertain functions are grouped into a new state that is, together with the other states of the system, estimated by a high-gain observer. With the estimated states a feedback is implemented that is based on the idea of linearization. Finally the proposed methodology is demonstrated for a two degree of freedom (DOF) robot manipulator and numerical results are presented.
Keywords: Robot synchronization, Synchronization, Robust synchronization.

1 Introduction

Synchronization is a phenomenon that has many examples in natural processes, such as the perfectly coincided oscillation of two pendulum clocks hanging from the same base [1], the synchronous firing of neurons [2],[3] or the symmetry of animal gaits [4]. As in these examples the synchronization is achieved by interconnections in the systems without any external interference, we speak of self-synchronization. Additionally we find numerous examples in different mechanical and electrical structures, such as transmitter receiver systems, quadruped robot movements [5] etc. where the synchronization is achieved by external inputs and couplings, because of which we speak of controlled synchronization. This article focuses on the controlled synchronization of robot manipulators. We find many applications in production processes, where the synchronous behavior of robotic systems is necessary for the production of parts with equal quality. In surgery,

* Corresponding Author, e-mail: gualberto.solis@cucei.udg.mx

© E. V. Cuevas, M. A. Perez, D. Zaldivar, H. Sossa, R. Rojas (Eds.)
Special Issue in Electronics and Biomedical Informatics,
Computer Science and Informatics
Research in Computing Science 35, 2008, pp. 63-73

new minimal invasive robotic systems have been developed [6] that require the synchronization of the robot with the trajectory that is generated by the operating surgeon.

While the control of robot manipulators is a classical control problem, the problem of synchronization of robots has not received much attention. We can find some approaches in [7] where the parameters of the system are estimated by an observer using only angular positions. Using those estimates an adaptive control strategy is realized. However, this technique requires the exact knowledge of the dynamics of the system, which results in a non robust approach. Therefore, in a realistic case, there are no knowledge of the frictions terms, parameter variations, etc.

In this article we assume that the parameters and the dynamics of the robot system are uncertain and that only the angular positions can be measured. Departing from the ideas presented in [8] we use the proposed robust nonlinear control scheme for the Multiple Input Multiple Output (MIMO) case. The methodology achieves the synchronization of an arbitrary number of robots in spite of structural and parametric differences of the robots and it is robust against external perturbations, friction and parameter variations. After a transformation of the system into a linearizable canonical form, the uncertain dynamics and parameters are lumped into an new state. This new state is, as well as the angular velocities, unknow and because of which it is estimated by a high-gain observer. With the estimated states a stabilizing controller is implemented that bases on the idea of linearization. Finally the robots are connected in a mutual pattern that achieves the synchronization between the robots and with respect to a trajectory that is given by the user.

2 Problem Statement

Let us consider a robotic manipulator that consist of w links and has m rotatory degrees of freedom that create the generalized angular positions $q_i, i = 1...m$. We assume that it is possible to generate m torques $\tau_i,\ \ i = 1...m$ in the link connections, for example with the help of electrical motors, hydraulic systems etc. It was presumed that it is possible to measure the angular positions of links at each point in time while the availability of the angular velocity was not postulated. The links of the robot were modeled as perfectly stiff, i.e. bending and vibration effects were neglected. With the help of the *Lagrange* or similar equations we can derive the following model of a robot with m rotatory degrees of freedom:

$$\ddot{q} = M(q)^{-1} \left(\tau - C(q,\dot{q})\dot{q} - g(q) - p(\dot{q})\right) \tag{1}$$

$M(q) \in R^{m\times m}$ is the symmetric, positive definite inertia matrix while $C(q,\dot{q})\dot{q} \in R^m$ represent the Coriolis and centrifugal forces. $g(q) = \frac{\partial}{\partial q}E_{pot} \in R^m$ denotes the gravity forces and the friction in the element connections is represented by the function $p(\dot{q}) \in R^m$. We decided to use the static friction model that was proposed by [12]. It is represented by the following equation:

$$p_i(\dot{q}_i) = B_{v_i}\dot{q}_i + B_{f_{i,1}}\left(1 - \frac{2}{1+e^{2\omega_{i,1}q_i}}\right) + B_{f_{i,2}}\left(1 - \frac{2}{1+e^{2\omega_{i,2}q_i}}\right) \quad i = 1...m \quad (2)$$

Where B_v is used to model the viscous friction while the remaining terms approximate the Coulomb and Stribeck friction effects.
We carry out the following transformation:

$$\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix} \; and \; \begin{bmatrix} x_{m+1} \\ \vdots \\ x_{2m} \end{bmatrix} = \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_m \end{bmatrix} \quad (3)$$

Now (1) becomes a nonlinear $m \times m$ MIMO system, that is characterized by $n = 2m$ first order differential equations:

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)\tau_1 + ... + g_m(x)\tau_m \\ y &= [x_1...x_m]^T \end{aligned} \quad (4)$$

With the states $x \in R^n$, the system input $\tau \in R^m$ and the system output $y \in R^m$. The system is characterized by the function $f(x) \in R^n$ and the matrix $g(x) \in R^{n\times n}$:

$$f(x) = \begin{bmatrix} x_{m+1} \\ \vdots \\ x_{2m} \\ M(x)^{-1}\left(-C(x,\dot{x})\dot{x} - g(x) - p(\dot{x})\right) \end{bmatrix}; g(x) = \begin{bmatrix} 0_{m\times m} & 0_{m\times m} \\ 0_{m\times m} & M(x)^{-1} \end{bmatrix} \quad (5)$$

For the synchronization of two or various robotic manipulators, we will presume that every system fulfills the following assumptions:

A.1: Only the angular positions $[q_1...q_m]$ can be measured at each point in time, i.e. not all the states x_i, $i = 1...n$ of the system are available.
A.2: There is no exact knowledge of the structure and the coefficients of $M(q)$, $C(q,\dot{q})$, $g(q)$ and $p(\dot{q})$.
A.3: The robotic manipulators may be strictly different, but they all have the same degrees of freedom and the same inputs.

There are numerous synchronization designs, such as serial or parallel master-slave models etc. [7]. However, in this work we will discuss the mutual synchronization pattern, where synchronous behavior is achieved with the interaction between the robots. The robots are arranged in a network and every robot could be connected to all the other robots. Let us suppose we have a number of l robots. For mutual synchronization the trajectories of reference $y_{ref_{i,k}}$ with

$i = 1...l$, $k = 1...m$ of the robot i for the degree of freedom k are calculated as follows:

$$y_{ref_{i,k}} = y_{d_k} - \sum_{j=1,j\neq i}^{l} K_{cp_{i,j}}(y_{i,k} - y_{j,k}) \tag{6}$$

Where $y_d \in R^m$ is the desired trajectory that is given by the user, which is equal for all the robots and has to be smooth. $K_{cp_{i,j}}$ are the so called coupling factors. They define how strong the robot i will interact with the robot j. High values of the coupling factors will lead to a fast synchronization between the robots, low values will lead to a fast synchronization of the robots with the desired trajectory y_d. The synchronization of all robot manipulators is achieved if $lim_{t\to\infty}\|y_{ref_{i,k}}(t) - y_{i,k}(t)\| \to 0$ for $i = 1...l$ and $k = 1...m$. It is straightforward that this is only possible if also $lim_{t\to\infty}\|y_{d_k}(t) - y_{i,k}(t)\| \to 0$ for $i = 1...l$ and $k = 1...m$. The synchronization problem can be formulated as the design of the interconnections between the robots and the creation of control feedbacks for the robots. In the next chapter we will propose a robust control feedback strategy that is well suited for the mutual synchronization of robots.

3 The robust synchronization scheme

For the implementation of the proposed feedback scheme the system has to be transformed on *Burnes Isidori Normal Form*. Because this transformation requires the knowledge of the relative degree vector, we will use the following definition [11]:

Definition 2: (Relative Degree) The relative degree vector $[r_1...r_m]$ of an affine MIMO system as in (4) is defined by:

1. $L_{g_j}L_f^k h_i(x) = 0$ for all x close to x_0 and $1 \leq i,\ j \leq m,\ 0 \leq k \leq r_i - 2$
2. The matrix $A(x_0)$ is nonsingular

$$A(x_0) = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x_0) & \dots & L_{g_m}L_f^{r_1-1}h_1(x_0) \\ \vdots & \ddots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x_0) & \dots & L_{g_m}L_f^{r_m-1}h_m(x_0) \end{bmatrix}$$

With this definition we can find that, for the robot manipulators $A(x) = M(x)^{-1}$ and that the relative degree of every input is $r_i = 2$. As $\zeta = r_1 + ... + r_m = n$ the system has full order and therefore it has no internal dynamics. Now we can carry out the transformation $z = \phi(x), \phi : R^n \to R^n$, with:

$$\phi(x) = \begin{bmatrix} z_{1,1} \\ z_{2,1} \\ z_{1,2} \\ \vdots \\ z_{2,m} \end{bmatrix} = \begin{bmatrix} h_1(x) \\ L_f h_1(x) \\ h_2(x) \\ \vdots \\ L_f h_m(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_{m+1} \\ x_2 \\ \vdots \\ x_{2m} \end{bmatrix} \tag{7}$$

With this transformation the system (4) is linearizable and becomes:

$$\dot{z} = \begin{bmatrix} \dot{z}_{1,1} \\ \dot{z}_{2,1} \\ \dot{z}_{1,2} \\ \vdots \\ \dot{z}_{2,m} \end{bmatrix} = \begin{bmatrix} z_{2,1} \\ \alpha_1(z) + \sum_{j=1}^{m} \beta_{j,1}(z)\tau_j \\ z_{2,2} \\ \vdots \\ \alpha_m(z) + \sum_{j=1}^{m} \beta_{j,m}(z)\tau_j \end{bmatrix} \tag{8}$$
$$y = [y_1 \ \dots \ y_m]^T = [z_{1,1} \ \dots \ z_{1,m}]^T$$

In the case of robot manipulators the transformation $z = \phi(x), \phi : R^n \to R^n$ is always a diffeomorphism and thereby $z = \phi(x)$ is an invertible transformation. This means, that if we can control (8) we can also control (4). The vector $\alpha(z) : R^{2m} \to R^m$ is defined by $\alpha_i(z) = L_f^2 h_i(\phi^{-1}(z))$ and the matrix $\beta(z) : R^{2m} \to R^{m\times m}$ by $\beta_{j,i}(z) = L_{g_j} L_f h_i(\phi^{-1}(z))$. We find that $\alpha(z) = [f_{m+1}(\phi^{-1}(z))...f_{2m}(\phi^{-1}(z))]^T$ and $\beta(z) = M(\phi^{-1}(z))^{-1}$. Thus the linearizing controller $\tau = \beta(z)^{-1}(v - \alpha(z))$ is called the perfect control. If we choose v_i for $i = 1...m$ as follows

$$\begin{aligned} v_i &= \dot{z}_{2,i} = \ddot{y}_i = \\ &\ddot{y}_{ref_i} - \rho_{1,i}(\dot{y}_i - \dot{y}_{ref_i}) - \rho_{2,i}(y_i - y_{ref_i}) \end{aligned} \tag{9}$$

the outputs of the system can follow any affine vector of trajectories of reference $y_{ref} \in C^2$ without any permanent error.

Remark 1: The controller $\tau = \beta(z)^{-1}(v - \alpha(z))$ requires the exact knowledge of all the states z_i as well as the knowledge of $\alpha_i(z) = L_f^2 h_i(x)$ and $\beta_{j,i}(z) = L_{g_j} L_f h_i(x)$ for $i = 1...m, j = 1...m$ at each point in time.

However, as we have assumed in assumption A.2, in the case of the robot manipulators we have no exact knowledge of the structure and the coefficients of $M(q)$, $C(q, \dot{q})$, $g(q)$ and $p(\dot{q})$ which means that also $\alpha(z)$ and $\beta(z)$ are uncertain. Besides, according to assumption A.1, only the angular positions $y = [z_{1,1}...z_{1,m}]^T = [q_1...q_m]^T$ can be measured while the angular velocities $\dot{y} = [z_{2,1}...z_{2,m}]^T = [\dot{q}_1...\dot{q}_m]^T$ are unknown.

Following the ideas that presented in [8], [9] and [10] where the controller requires only least prior knowledge about the system (8) and can stabilize the system at the origin or make it follow any affine trajectory. The control scheme does not require the knowledge of $\alpha(z)$ and $\beta(z)$. The idea is to lump these uncertain terms into a new observable state that can be reconstructed from the available angular positions $[q_1...q_m]$. We introduce the new variable vector $\Theta \in R^m$, which contains the uncertain functions $\alpha(z)$ and $\beta(z)$ for $i = 1...m$:

$$\Theta_i(z, \tau) = \alpha_i(z) + \sum_{j=1}^{m} \left(\beta_{j,i}(z) - \beta_{e_{j,i}}(z)\right)\tau_j \tag{10}$$

$\beta_e(z) \in R^{m\times m}$ is an user-defined approximation of $\beta(z)$ that has to fulfill $sign(\beta_e(z)) = sign(\beta(z))$. With this we can rewrite the system (8), for $i = 1...m$:

$$\begin{aligned} \dot{z}_{1,i} &= z_{2,i} \\ \dot{z}_{2,i} &= \Theta_i(z,\tau) + \textstyle\sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j \end{aligned} \tag{11}$$

Now we augment our system by m additional states $\eta_i(t) = \Theta_i(z,\tau)$ with $i = 1...m$. In this way (11) becomes:

$$\begin{aligned} \dot{z}_{1,i} &= z_{2,i} \\ \dot{z}_{2,i} &= \eta_i(t) + \textstyle\sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j \\ \dot{\eta}_i(t) &= \Xi_i(z,\eta,\tau) \end{aligned} \tag{12}$$

Where $\Xi_i(z,\eta,\tau) = \frac{\partial\Theta_i}{\partial z}\frac{dz}{dt} + \frac{\partial\Theta_i}{\partial\tau}\frac{d\tau}{dt}$, in assumption A.1 we have supposed that we have no exact knowledge about all the states x_i. Consequently the new state vector $\eta(t) \in R^m$ is also unknown. To solve this problem, we construct the following high-gain observer that is based on the available states $y = [z_{1,1}...z_{1,m}]$.

$$\begin{aligned} \dot{\hat{z}}_{1,i} &= \hat{z}_{2,i} + L\kappa_{1,i}(z_{1,i} - \hat{z}_{1,i}) \\ \dot{\hat{z}}_{2,i} &= \hat{\eta}_i + \textstyle\sum_{j=1}^{m} \beta_{e_{j,i}}(z)\tau_j + L^2\kappa_{2,i}(z_{1,i} - \hat{z}_{1,i}) \\ \dot{\hat{\eta}}_i &= L^3\kappa_{3,i}(z_{1,i} - \hat{z}_{1,i}), \quad i = 1...m \end{aligned} \tag{13}$$

Now we have to choose the coefficients $\kappa_{i,j}$ in such a way that the polynomials $s^3 + \kappa_{1,i}s^2 + \kappa_{2,i}s + \kappa_{3,i}, \quad i = 1...m$ have poles with negative real parts. L is a tuning parameter that has a strong influence on the error dynamics. Based on the estimates of the uncertainties $\eta(t)$ and the estimates of $[z_{2,1}...z_{2,m}]^T$ we can construct the following linearizing-like feedback controller

$$\tau = \beta_e(z)^{-1}(v - \hat{\eta}) \tag{14}$$

With the input vector $v \in R^m$ that is defined as:

$$\begin{aligned} v_i &= \ddot{y}_{ref_i} - \rho_{1,i}(\hat{z}_{2,i} - \dot{y}_{ref_i}) - \rho_{2,i}(z_{1,i} - y_{ref_i}), \\ i &= 1...m \end{aligned} \tag{15}$$

Proposition 1: The robust feedback method consists of the dynamic estimator (13) and the linearizing controller (15), that was constructed using the estimates of Θ (i.e. $\eta(t)$) and z that are provided by the high-gain observer.

Proof The proof of stability is equal for all the m degrees of freedom. Because of this, we will carry out a parallel proof for all the degrees of freedom and $i = 1...m$ will be valid. The stability of the observer, we define an estimation error $e_i \in R^3$ in the following way: $e_{j,i} = L^{r-j+1}(z_{j,i} - \hat{z}_{j,i}), \ j = 1, 2$ and $e_{3,i} = \eta_i - \hat{\eta}_i$. Now, using (12) and (13) we can write the error dynamics $\dot{e}_i$ as:

$$\begin{aligned} \dot{e}_{1,i} &= L(-\kappa_{1,i}e_{1,i} + e_{2,i}) \\ \dot{e}_{2,i} &= L(-\kappa_{2,i}e_{1,i} + e_{3,i}) \\ \dot{e}_{3,i} &= -L\kappa_{r+1,i}e_{1,i} + \Xi_i \end{aligned} \tag{16}$$

Or written in Matrix form:

$$\dot{e}_i = L \underbrace{\begin{bmatrix} -\kappa_{1,i} & 1 & 0 \\ -\kappa_{2,i} & 0 & 1 \\ -\kappa_{3,i} & 0 & 0 \end{bmatrix}}_{A_i(\kappa)} e_i + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \Xi_i \end{bmatrix}}_{\Gamma_i} \tag{17}$$
$$= LA_i(\kappa)e_i + \Gamma_i$$

The matrix $A_i(\kappa)$ is Hurwitz if the poles of the polynomial $s^3+\kappa_{1,i}s^2+\kappa_{2,i}s+\kappa_{3,i}$ are in the left pane of the complex plane. If this is the case, then, according to Lyapunov, there exists a positive definite and symmetric matrix P_i such that $P_iA_i + A_i^TP_i = -I_n$ where I_n is the identity matrix of dimension n. Now we choose $V_i(e_i) = e_i^TP_ie_i$ as Lyapunov function and get:

$$\dot{V}_i(e_i) = \frac{\partial V_i(e_i)}{\partial e_i}\dot{e}_i = -L\|e_i\|^2 + 2e_i^TP_i\Gamma_i \tag{18}$$
$$\leq -L\|e_i\|^2 + 2\|P_i\|\|e_i\|\|\Gamma_i\|$$

If Γ_i satisfies $\|\Gamma_i\| < r_1$ and $\|e_i\| < r_2$ for some $r_1 > 0$ and $r_2 > 0$ then $\|P_i\|\|e_i\|\|\Gamma_i\|$ is a bounded function. Let $\mu_i > 0$ be some positive constant and $2\|P_i\|\|e_i\|\|\Gamma_i\| < \mu_i$. We can write $\dot{V}_i(e_i) \leq -L\|e_i\|^2 + \mu_i$ for the stability of the observer $\|e_i\| \leq \sqrt{\frac{\mu_i}{L}}$ has to be fulfilled for all i. We can see, that the estimation error e_i depends directly on L. As L increases, e_i will decrease and thereby also the estimation error bound. Because of this, L should be chosen as big as possible. We conclude: As all Γ_i are bounded, if $L > L^* > 0$ then $e(t) \to 0$ for $t \to \infty$ and $(\hat{z}, \hat{\eta}) \to (z, \eta)$. With this we conclude, that (13) and (14) yield asymptotical stabilization of the system (8).□

To illustrate the proposed control scheme we will now apply the methodology to the case of a robot manipulator with $m = 2$ rotatory degrees of freedom. With the help of the *Lagrange* or similar equations we can calculate $M(q), C(q, \dot{q}), g(q)$ of (1) as follwos

$$\begin{aligned}
M_{11} &= m_1l_{c1}^2 + m_2l_1^2 + m_2l_{c2}^2 + I_1 + I_2 + 2m_2l_1l_{c2}cos(q_2)\\
M_{12} &= m_2l_{c2}^2 + m_2l_1l_{c2}cos(q_2) + I_2\\
M_{21} &= m_2l_{c2}^2 + m_2l_1l_{c2}cos(q_2) + I_2\\
M_{22} &= m_2l_{c2}^2 + I_2\\
C_{11} &= -m_2l_1l_{c2}sin(q_2)\dot{q}_2\\
C_{12} &= -m_2l_1l_{c2}sin(q_2)(\dot{q}_1 + \dot{q}_2)\\
C_{21} &= m_2l_1l_{c2}\dot{q}_1sin(q_2)\\
C_{22} &= 0\\
g_1(q) &= gsin(q_1(m_1l_{c1} + m_2l_1) + m_2gsin(q_1 + q_2)l_{c2}\\
g_2(q) &= m_2gl_{c2}sin(q_1 + q_2)
\end{aligned}$$

We will use the same friction term $p(\dot{q}) \in R^2$ as in (2). Again $q \in R^2$ are the angular positions of the links while $\dot{q} \in R^2$ are the angular velocities and $\tau \in R^2$ are the torques that are applied to the links. l_1, l_2 are the lengths of the links and l_{c1}, l_{c2} are the distances to their centers of mass. m_1, m_2 are the masses of the two elements, I_1, I_2 are their moments of inertia (including the motors, joints etc.) and g is the acceleration of gravity. After replacing $[x_1, x_2]^T = [q_1, q_2]^T$, $[x_3, x_4]^T = [\dot{q}_1, \dot{q}_2]^T$ and $M^*(x) = M^{-1}(x)$ we can rewrite our system (1):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ f_3(x) + M^*_{1,1}\tau_1 + M^*_{1,2}\tau_2 \\ f_4(x) + M^*_{2,1}\tau_1 + M^*_{2,2}\tau_2 \end{bmatrix} \tag{19}$$

Where $[f_3(x), f_4(x)]^T = M(x)^* \left(-C(x,\dot{x})\dot{x} - g(x) - p(\dot{x})\right)$. Using Definition 2 we find that the relative degree is $r = 2$. Now we introduce the augmented state vector $\eta(t) = [\eta_1(t), \eta_2(t)]^T$ and the user-defined approximation of $\beta_e(z)$ of $\beta(z) = M(\phi^{-1}(z))^{-1}$ and get:

$$\begin{aligned} \dot{z}_{1,1} &= z_{2,1} \\ \dot{z}_{2,1} &= \eta_1(t) + \beta_{e_{1,1}}(z)\tau_1 + \beta_{e_{2,1}}(z)\tau_2 \\ \dot{\eta}_1(t) &= \Xi_1(z,\eta,\tau) \\ \dot{z}_{1,2} &= z_{2,2} \\ \dot{z}_{2,2} &= \eta_2(t) + \beta_{e_{1,2}}(z)\tau_1 + \beta_{e_{2,2}}(z)\tau_2 \\ \dot{\eta}_2(t) &= \Xi_2(z,\eta,\tau) \end{aligned} \tag{20}$$

For the reconstruction of the angular velocities $[z_{2,1}, z_{2,2}]^T$ and the extended state $\eta(t)$ we construct the high-gain observer:

$$\begin{aligned} \dot{\hat{z}}_{1,1} &= \hat{z}_{2,1} + L\kappa_{1,1}(z_{1,1} - \hat{z}_{1,1}) \\ \dot{\hat{z}}_{2,1} &= \hat{\eta}_1 + \textstyle\sum_{j=1}^{2} \beta_{e_{j,1}}(z)\tau_j + L^2\kappa_{2,1}(z_{1,1} - \hat{z}_{1,1}) \\ \dot{\hat{\eta}}_1 &= L^3\kappa_{3,1}(z_{1,1} - \hat{z}_{1,1}) \\ \dot{\hat{z}}_{1,2} &= \hat{z}_{2,2} + L\kappa_{1,2}(z_{1,2} - \hat{z}_{1,2}) \\ \dot{\hat{z}}_{2,2} &= \hat{\eta}_2 + \textstyle\sum_{j=1}^{2} \beta_{e_{j,2}}(z)\tau_j + L^2\kappa_{2,m}(z_{1,2} - \hat{z}_{1,2}) \\ \dot{\hat{\eta}}_2 &= L^3\kappa_{3,2}(z_{1,2} - \hat{z}_{1,2}) \end{aligned} \tag{21}$$

With the estimates of (21) we can implement the following controller:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \beta_{e_{1,1}}(z) & \beta_{e_{1,2}}(z) \\ \beta_{e_{2,1}}(z) & \beta_{e_{2,2}}(z) \end{bmatrix}^{-1} \begin{bmatrix} v_1 - \hat{\eta}_1 \\ v_2 - \hat{\eta}_2 \end{bmatrix} \tag{22}$$

In order to follow the smooth trajectory of reference $y_{ref} = [y_{ref_1}, y_{ref_2}]^T$ we choose $v = [v_1, v_2]^T$ as:

$$\begin{aligned} v_1 &= \ddot{y}_{ref_1} - \rho_{1,1}(\hat{z}_{2,1} - \dot{y}_{ref_1}) - \rho_{2,1}(z_{1,1} - y_{ref_1}) \\ v_2 &= \ddot{y}_{ref_2} - \rho_{1,2}(\hat{z}_{2,2} - \dot{y}_{ref_2}) - \rho_{2,2}(z_{1,2} - y_{ref_2}) \end{aligned} \tag{23}$$

Now the coupling factors were chosen as $K_{cp} = 10$ while the we consider arbitrary initial conditions for q_{1i}, q_{2i} and $\dot{q}_{1i}, \dot{q}_{2i}$. The controller was switched on after 5 seconds and after 10 seconds a perturbation torque $\tau_{pert} = 10\ Nm$ was applied to both link connections of all the robots. τ_{pert} was turned off after 15 seconds. For the trajectory $y_d \in R^2$ we chose an arbitrary smooth function. The following variables were chosen equally for all four robots:

L	g	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
20	9.81	0.35	0.3	0.175	0.145	0.0064	0.004

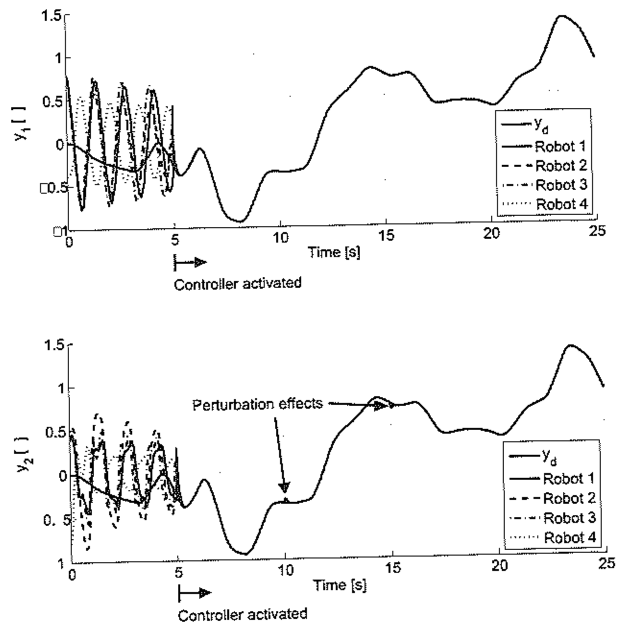
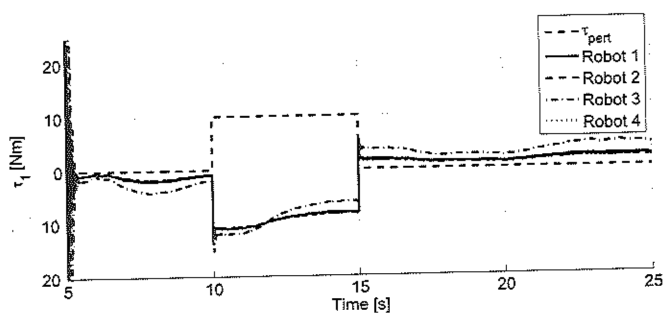


Fig. 1. (a) y_{d_1} and system output y_1, (b) y_{d_2} and system output y_2

In Fig. 1 we can see the free oscillation of the uncontrolled robots in the first 5 seconds. After this period the robots follow the arbitrary, user given trajectory y_d with actually very small errors. In Fig. 2 one can identify the torques generated to compensate the perturbation. Indeed, note that in Fig. 1, the effect of the



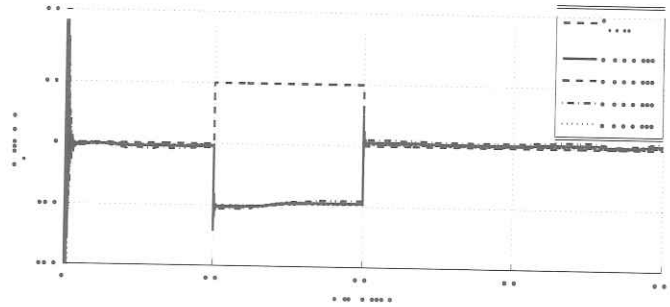


Fig. 2. (a) τ_{pert_1} and system input τ_1, (b) τ_{pert_2} and system input τ_2

perturbation is actually very low (after 5 and after 10 seconds). This shows that the approach does not require the estimation of the perturbation.

4 Conclusions

In this work we have presented a robust control scheme that achieves synchronization of robot manipulators with an arbitrary number of degrees of freedom. It compensates unmodeled dynamics, uncertain or time-varying parameters as well as external perturbations and requires only the measurement of the angular positions at each point in time. The central feature of this approach is that the uncertainties are lumped into an extended state, which is reconstructed by a high-gain observer. Based on this estimation a linearizing-like control law is implemented that achieves the synchronization in combination with a mutual connection pattern of the robots. The methodology was demonstrated for the case of a 2 DOF robot manipulator and validated by numerical results. The proposed control scheme can also be applied to other mechanical systems, such as robot manipulators with linear degrees of freedom and in combination with other synchronization patterns.

References

[1] A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization - A universal concept in nonlinear sciences*, Cambridge University Press, Cambrige, 2001

[2] G. Renversez, *Synchronization in two neurons: Results for a two-component dynamical model with time-delayed inhibition*, Physica D 114 p. 147-171, 1998

[3] B. Lading, E. Mosekilde, S. Yanchuk, Y. Maistrenko, *Chaotic Synchronization between Coupled Pancreatic β-Cells*, Prog. Theor. Phys. 139, p.164-177, 2000

[4] J.J. Collins, I.N. Stewart, *Coupled Nonlinear Oscillators and the Symmetries of Animal Gaits*, Nonlinear Science Vol.3, p. 349-392, 1993

[5] H. Kimura, K. Takase, *Adaptive Running of a Quadruped Robot Using Forced Vibration and Synchronization*,Journal of Vibration and Control 2006, May 2006

[6] J.W. Hills, J.F. Jensen, *Telepresence technology in medicine: principles and applications*, Proceedings of the IEEE, Vol. 86, p.569-580, March 1998

[7] H. Nijmeijer, A. Rodríguez-Angeles, *Synchronization of Mechanical Systems*, World Scientific Publishing, 2003

[8] R. Femat, J. Alvarez-Ramírez, G. Fernández-Anaya, *Adaptive synchronization of high-order chaotic systems: a feedback with low-order parametrization*, Physica D 139 p. 231-246, 1999

[9] R. Femat, R. Jauregui-Ortíz, G. Solís-Perales, *A Chaos-Based Communication Scheme via Robust Asymptotic Feedback*, IEEE Transactions on Circuits and Systems 1, Vol.48, p. 1161-1169, October 2001

[10] G. Solís-Perales, S. Bowong, R. Femat, *Synchronization of Dynamical Systems With Differente Order And Topology*, 1st IFAC Conference on Analysis and Control of Chaotic Systems, p.175-180, June 2006

[11] A. Isidori *Nonlinear Control Systems*, Springer Verlag, Berlin, 1989

[12] R. Hensen, G. Angelis, M. Molengraft, A. Jaeger, J. Kok *Grey-box modeling of friction: An experimental case-study*, European Journal of Control, 6, p. 258-267, 2000